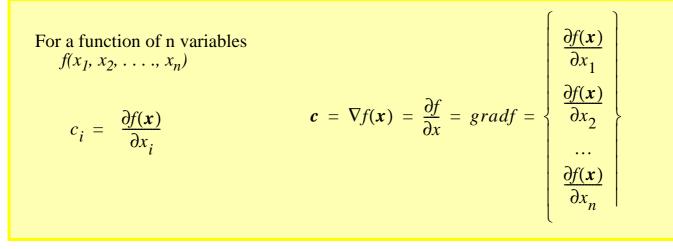
Optimum Design Concepts

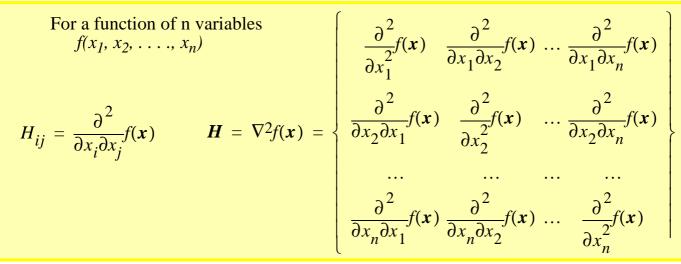
- Methods used for design optimization does not depend on the field of engineering.
- Broad classification of the optimization Optimality criteria methods (indirect methods) Search methods (direct search methods)
- Optimality criteria are the conditions a function must satisfy at its minimum point.
- Study of optimality conditions are necessary regardless of the method of optimization used.

• Gradient Vector (the vector of first derivatives):



•Geometrically, the gradient vector at point x^p is normal to the tangent plane to the function at that point, and points in the direction of maximum increase in the function.

• Hessian Matrix (the matrix of second derivatives):



•Hessian matrix is always a symmetric matrix

$$\frac{\partial^2}{\partial x_i \partial x_j} f = \frac{\partial^2}{\partial x_j \partial x_i} f$$

- Taylor Series Expansion:
 - •for a function with one variable, Taylor series expansion about x^p

$$f(x) = f(x^{p}) + \frac{df(x^{p})}{dx}(x - x^{p}) + \frac{1}{2}\frac{d^{2}f(x^{p})}{dx^{2}}(x - x^{p})^{2} + R$$

At a small distance *d* from x^{p}
 $x = x^{p} + d$
$$f(x^{p} + d) = f(x^{p}) + \frac{df(x^{p})}{dx}d + \frac{1}{2}\frac{d^{2}f(x^{p})}{dx^{2}}d^{2} + R$$

•for a function with *n* variables

$$f(\boldsymbol{x}) = f(\boldsymbol{x}^{p}) + \nabla f(\boldsymbol{x}^{p})^{T} \bullet (\boldsymbol{x} - \boldsymbol{x}^{p}) + \frac{1}{2}(\boldsymbol{x} - \boldsymbol{x}^{p})^{T} \bullet \boldsymbol{H}(\boldsymbol{x}^{p}) \bullet (\boldsymbol{x} - \boldsymbol{x}^{p}) + \boldsymbol{R}$$

•change in the value of the function in moving from x^p to a neighboring point $d = x - x^p$ distance away from it

$$\Delta f = f(\boldsymbol{x}^{p} + \boldsymbol{d}) - f(\boldsymbol{x}^{p}) = \nabla f(\boldsymbol{x}^{p})^{T} \bullet \boldsymbol{d} + \frac{1}{2}\boldsymbol{d}^{T} \bullet \boldsymbol{H}(\boldsymbol{x}^{p}) \bullet \boldsymbol{d} + R$$

September 6, 1999

- Quadratic Forms and Definite Matrices:
 - •Quadratic form is a special nonlinear function having only secondorder terms

For example;
$$F(\mathbf{x}) = x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_2x_3 + 2x_3x_3$$

Representations $F(\mathbf{x}) = \frac{1}{2}$ $p_{ij}x_ix_j = \frac{1}{2}\mathbf{x}^T \mathbf{P} \mathbf{x}$
 $i = 1j = 1$

•Every quadratic form can be put into the following form with a symmetric A matrix

$$F(\mathbf{x}) = \frac{1}{2}\mathbf{x}^{T}\mathbf{P}\mathbf{x} = \frac{1}{2}\mathbf{x}^{T}\mathbf{A}\mathbf{x}$$
 $a_{ij} = \frac{1}{2}(p_{ij} + p_{ji})$

- •Many matrices can be associated with a quadratic function, all of them are asymmetric. There is only one unique symmetric matrix.
- •The symmetric A matrix determines the nature of the quadratic form.

- For a given value of **x** a quadratic form $F(\mathbf{x}) = 1/2 \mathbf{x}^T A \mathbf{x}$ may be either positive, negative, or zero
 - A quadratic form is called positive definite if $x^T A x$ is always positive except for F(0).
 - It is called negative definite if $x^T A x < 0$ for all x except x = 0.
 - If a quadratic form is $x^T A x \ge 0$ for all x and there exists one nonzero $x \ (x \ne 0)$ with $x^T A x = 0$, then it is called positive semidefinite.
 - If a quadratic form is $x^T A x \le 0$ for all x and there exists one nonzero $x \ (x \ne 0)$ with $x^T A x = 0$, then it is called positive semidefinite.
 - A quadratic form which is positive for some vectors **x** and negative for others is called indefinite.

• Check the eigenvalues of the symmetric $n \times n A$ matrix associated with the quadratic form $F(x) = 1/2 x^T A x$.

•*F*(\boldsymbol{x}) is positive definite if and only if all eigenvalues of \boldsymbol{A} are strictly positive, i.e. $\lambda_i > 0$, i = 1 to n.

• $F(\mathbf{x})$ is positive semidefinite if and only if all eigenvalues of \mathbf{A} are non-negative, i.e. $\lambda_i \ge 0$, i = 1 to n.

•*F*(*x*) is negative definite if and only if all eigenvalues of *A* are strictly negative, i.e. $\lambda_i < 0$, i = 1 to n.

• $F(\mathbf{x})$ is negative semidefinite if and only if all eigenvalues of \mathbf{A} are non-positive, i.e. $\lambda_i \leq 0$, i = 1 to n. • $F(\mathbf{x})$ is indefinite if some $\lambda_i < 0$ and some other $\lambda_i > 0$.